

Optimal control with discontinuous running cost: eikonal equation and shape-from-shading

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Abstract

We consider a class of Hamilton-Jacobi equations with discontinuous coefficients that contains as examples the eikonal equation with discontinuous refraction index, or the Hamilton-Jacobi equation of shape-from-shading with discontinuous intensity function. In general the Dirichlet problem for such equations does not have unique solution, however we can characterize the minimal and maximal solution and, under appropriate compatibility conditions, prove existence of a unique solution in the sense of viscosity solutions.

1 Introduction

In this paper we want to discuss the following Dirichlet problem for the Bellman equation of optimal control

$$\begin{aligned} \lambda u(x) + \sup_{a \in A} \{-f(x, a) - Du(x) \cdot h(x, a)\} &= g(x), \\ &x \in \Omega \\ u(x) = 0, &x \in \partial\Omega \end{aligned} \quad (1.1)$$

where Ω is an open subset of $\mathbb{R}^N$, $\lambda \geq 0$ and $g : \Omega \rightarrow [0, +\infty[$ will always be locally bounded and Borel measurable, hence possibly discontinuous. In the study of (1.1) it is fundamental to consider the following control system

$$\begin{cases} \dot{y}(t) = f(y(t), \alpha(t)) \\ y(0) = x \end{cases} \quad (1.2)$$

where $f : \mathbb{R}^N \times A \rightarrow \mathbb{R}^N$ is a continuous function and A is a compact subset of a metric space. In order to simplify the assumptions and avoid some technical difficulties, the set of admissible controls A in (1.2) is chosen to be the set of relaxed controls, i.e.

$$A = \{\alpha(\cdot) : \alpha : \mathbb{R}_+ \rightarrow P(D) \text{ measurable}\},$$

and $P(D)$ is the set of Radon probability measures on $D \subset \mathbb{R}^N$ a compact set (see the book by Warga [11]). Optimal control problems for the system (1.2) will be defined with respect to a running cost function $h : \mathbb{R}^N \times A \rightarrow \mathbb{R}$ and the functional

$$J(t, x, \alpha) = \int_t^\infty e^{-\lambda s} [h(y_\alpha(s), \alpha(s)) + g(y_\alpha(s))] ds. \quad (1.3)$$

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In the following $h : \mathbb{R}^N \times A \rightarrow [0, +\infty[$ will be supposed to be at least continuous. In fact, some more stringent assumptions are needed on the data h, f , and throughout the paper we will suppose that, for all $R > 0$, they satisfy

$$\begin{cases} |f(x, a) - f(y, a)| \leq L|x - y|, & x, y \in \mathbb{R}^N, a \in A, \\ |h(x, a) - h(y, a)| \leq L_R|x - y|, & |x|, |y| \leq R, a \in A, \end{cases} \quad (1.4)$$

where L, L_R, C are positive constants. As a consequence, for given $a \in A$ and $x \in \mathbb{R}^N$, there is a unique global solution of (1.2), that we denote by $y_\alpha(t; x) \equiv y(t)$.

A motivation for studying equation (1.1) with discontinuous coefficient g can be found in the applications. For instance in geometric optics, to describe the propagation of light, the eikonal equation appears

$$\sum_{i,j=1}^N a_{ij}(x) u_{x_i x_j}(x) = g(x),$$

where $a = \sigma \sigma^t$ and g has the meaning of the refraction index of the medium. As well known, refraction law applies across surfaces of discontinuity of g . Another example can be found in image processing and the shape-from-shading model. In this case we come up with the equation

$$\sqrt{1 + |Du(x)|^2} = \frac{1}{I(x)} (\equiv g(x))$$

and the object to reconstruct is the graph of the unknown function u . In this case $I(x) \in]0, 1]$ represents the intensity of light reflected by the object and it is discontinuous when the object has ledges. With a bit of convex analysis, both equations above can be rewritten in the form (1.1). Further motivation appears directly in control theory when discontinuous functions are used to represent targets (in this case g will be a characteristic function) or state constraints (when g is instead an indicator functions).

Being nonsmooth in general, solutions of (1.1) will be defined in the sense of the theory of viscosity solutions. An important point to analyze then comes up. Indeed with this notion of solution, when an equation has a discontinuous coefficient, we cannot interpret g in a pointwise sense. This is due to the fact that value functions

Discontinuous Optimal Control

Shasha Hu



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